Technical Appendices

A Diagrams

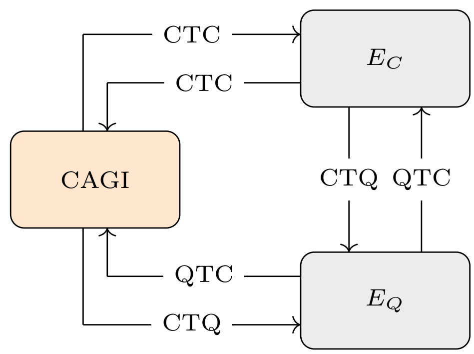

Fig. 1. Classical agent (CAGI) interacting via CTC, CTQ or QTC maps with classical E_C or quantum E_Q environments

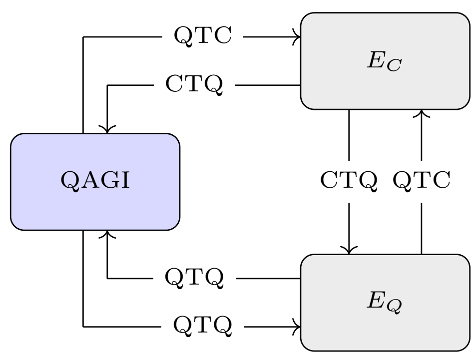

Fig. 2. Quantum agent (QAGI) interacting via QTC, CTQ or QTQ maps.

B Hamiltonian Dynamics

Classical Evolution Evolution is described by Hamilton's equations:

$$\dot{q}_i = \frac{\partial H_C}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H_C}{\partial q_i}. \tag{11}$$

This can be expressed more abstractly using the Poisson bracket. For two observables f, g, their Poisson bracket is:

$$\{f, g\}_{PB} = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right). \tag{12}$$

The time evolution of any observable f is then given by $\dot{f} = \{f, H_C\}_{PB}$. If $\{f, g\}_{PB} = 0$ (Poisson bracket), the observables f and g are said to commute, implying they can, in principle, be simultaneously determined with arbitrary precision. Classical logic and computation often implicitly rely on this property: the truth value of one proposition or the state of one register does not inherently interfere with another, distinct one unless explicitly coupled by H_C. For a classical AGI, H_C we model Hamiltonians as decomposable: $H_C = \sum_k H_{C,k}$, where each $H_{C,k}$ represents a functional aspect like learning (e.g., gradient descent dynamics [15]), reasoning (e.g., energy function of a Hopfield network or constraint satisfaction), or interaction. The commutativity of these underlying processes, or the variables they act upon, defines the classical computational semantics. In information theoretic terms, classical mechanical dynamics of CAGI can be expressed as follows. Let $\{\mathcal{R}_i\}_{i=1}^n$ be the classical registers of the agent, each described by a *commutative* von Neumann algebra $\mathcal{V}_i = L^\infty(\Omega_i, \mu_i)$. A micro-state of the whole agent–environment system is therefore a point $(\mathbf{q}, \mathbf{p}) \in \mathcal{M} = T^*\mathcal{C}$ with

$$q_i := X_i(\omega_i), \qquad p_i := M_i\, \dot{X}_i(\omega_i),$$

where $X_i \in \mathcal{V}_i$ is the random variable realised by register $\mathcal{R}_i$ and M_i is an information-theoretic weight term (e.g. an inverse learning rate or buffer capacity). The classical Hamiltonian functional $H_C\colon \mathcal{M} \to \mathbb{R}$ involves coordinates:

$$\dot{q}_i \;=\; \frac{\partial H_C}{\partial p_i}, \qquad \dot{p}_i \;=\; -\frac{\partial H_C}{\partial q_i}, \tag{13}$$

but now (13) is understood to act on *probability densities* $f_t(\mathbf{q}, \mathbf{p})$ pushed forward by the CTC channel $\mathrm{CTC}_t : L^\infty(\Omega) \to L^\infty(\Omega)$. For any pair of observables $f, g \in \bigoplus_i \mathcal{V}_i$ we retain the Poisson bracket:

$$\{f, g\}_{\mathrm{PB}} \;=\; \sum_{i=1}^{n} \Big(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \Big),$$

so the time derivative of f is $\dot{f} = \{f, H_C\}_{\mathrm{PB}}$. In information terms $\{f, g\}_{\mathrm{PB}} = 0$ *iff* the corresponding *classical channels* commute.

B.1 Quantum Hamiltonian dynamics

When transitioning to a quantum substrate, the AGI's state is described by a vector $|\psi\rangle$ in a Hilbert space $\mathcal{H}$ (or a density operator ρ acting on $\mathcal{H}$). Observables are represented by self-adjoint operators A acting on $\mathcal{H}$. The dynamics are governed by the Schrödinger equation:

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} |\psi(t)\rangle = H_Q |\psi(t)\rangle, \quad \text{or for density operators, } i\hbar \frac{\mathrm{d}}{\mathrm{d}t} \rho(t) = [H_Q, \rho(t)], \tag{14}$$

where H_Q is the quantum Hamiltonian operator and $[A, B] = AB - BA$ is the commutator. The set of bounded operators on $\mathcal{H}$ forms a C*-algebra, and the set of all observables can be considered within a von Neumann algebra $\mathcal{B}(\mathcal{H})$ (the algebra of bounded operators on $\mathcal{H}$). The key algebraic difference from the classical case lies in the non-commutativity of operators. If $[A, B] \neq 0$, the observables A and B are generally incompatible, meaning they cannot be simultaneously measured or defined with arbitrary precision (Heisenberg uncertainty principle). This non-commutativity is fundamental and has profound consequences which may be a constraint or benefit.

For a quantum AGI, the total Hamiltonian $H_Q = \sum_k H_{Q,k}$ would similarly consist of generators for different AGI functions. However, these $H_{Q,k}$ are now operators, and their mutual commutation relations, as well as their commutation with other relevant observables, dictate the AGI's behavior. For example, if a learning operator $H_{Q,learn}$ does not commute with a sensing operator $H_{Q,sens}$ representing environmental perception, then the act of learning can be disturbed by observation, and vice-versa, in a way that has no classical parallel. This non-commutative structure underpins quantum phenomena like entanglement and contextuality, which could be resources or challenges for a QAGI.

B.2 Quantum information formulation

In quantum information terms, transitioning to quantum involves replacing every classical register $\mathcal{V}_i = L^\infty(\Omega_i)$ by a *non-commutative* von Neumann algebra $\mathcal{V}_i = B(\mathcal{H}_i)$ acting on a Hilbert space $\mathcal{H}_i$. The full agent–environment is reflected by the tensor algebra $\mathcal{V} = \bigotimes_i \mathcal{V}_i \subseteq \mathcal{B}(\mathcal{H})$, with $\mathcal{H} = \bigotimes_i \mathcal{H}_i$. States are represented by density operators $\rho \in \mathcal{D}(\mathcal{H}) = \{\rho \geq 0,\ \operatorname{Tr}\rho = 1\}$, and a observable is an element $A \in \mathcal{V}$. When the evolution is closed and reversible the channel on $\mathcal{V}$ is the adjoint action of a unitary U_t:

$$\Phi_t^{(\mathrm{u})}(A) = U_t^\dagger A U_t, \qquad U_t = \exp\bigl(-\tfrac{i}{\hbar} H_Q t\bigr),$$

where the *Hamiltonian operator* $H_Q \in \mathcal{V}$ is the quantum analogue of H_C. In Schrödinger form this yields the familiar

$$i\hbar\dot{\rho}(t) = \bigl[H_Q, \rho(t)\bigr], \tag{15}$$

which is the generator $\mathcal{L}_{H_Q} = -\frac{i}{\hbar}[H_Q, \cdot]$ of a one-parameter group of QTC channels. Realistic AGI modules interact with—and are monitored by—their environment, so the fundamental dynamical object is a quantum channel $\Phi_t = \exp(t\mathcal{L})$, with Lindblad superoperator:

$$\mathcal{L}(\rho) = -\frac{i}{\hbar}[H_Q, \rho] + \sum_\alpha \Bigl(L_\alpha \rho L_\alpha^\dagger - \tfrac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\Bigr) \tag{16}$$

where the L_α's represent QTC measurement-and-feedback registers).